# Mathematical Model for Isobaric Energy Recovery Devices

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## Abstract

The use of isobaric energy recovery devices (isobaric ERDs) in seawater desalination plants has become widespread around the world. As a result, in recent years a greater variety of isobaric ERDs have been used, increasing the difficulties associated with the comparison of the reverse osmosis plant parameters. Therefore, the use of new isobaric ERDs has complicated calculation processes, plant recovery rate is now not always equal to Reverse Osmosis recovery rate. It has become necessary to develop new calculation approaches. Here, we proposed a mathematical model, based on normalized operation parameters, which allows the calculation of RO plant parameters such as flows, pressures and recovery.

This model will aid in achieving optimal performance and design of desalination plants using a variety of isobaric ERDs.

# I. INTRODUCTION

The desalination market is a growing market in constant evolution. Reverse osmosis seawater desalination plants are currently being built all over the world. In recent years, a large variety of new equipments, processes, chemical products and treatments have appeared. All of these new elements must be included into the plant design However, this inclusion requires a clear understanding of how these new technologies work and a normalization of performance parameters.

Currently, a variety of energy recovery technologies are used in RO plants: isobaric ERDs, Pelton turbines, Francis turbine and hydraulic turbocharger. The most widely used in new large SWRO desalination plants are Pelton Turbines and isobaric ERDs.

Figure 1 illustrates a typical reverse osmosis process with Pelton Turbine for brine energy recovery

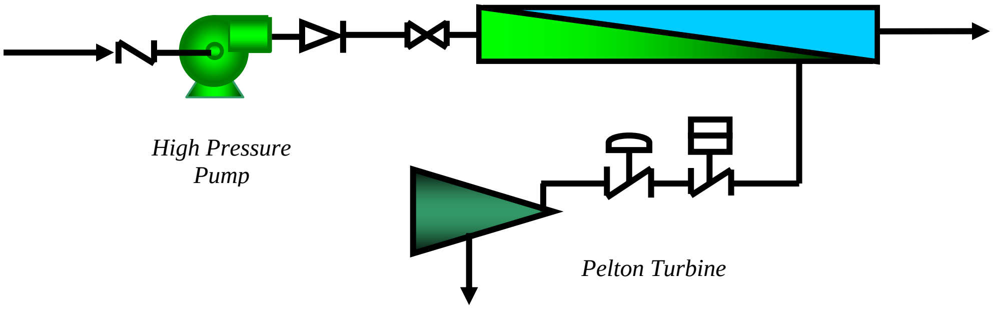


Figure 1. Reverse Osmosis with Pelton Turbine

Figure 2 illustrates a typical reverse osmosis process with isobaric ERDs for brine energy recovery

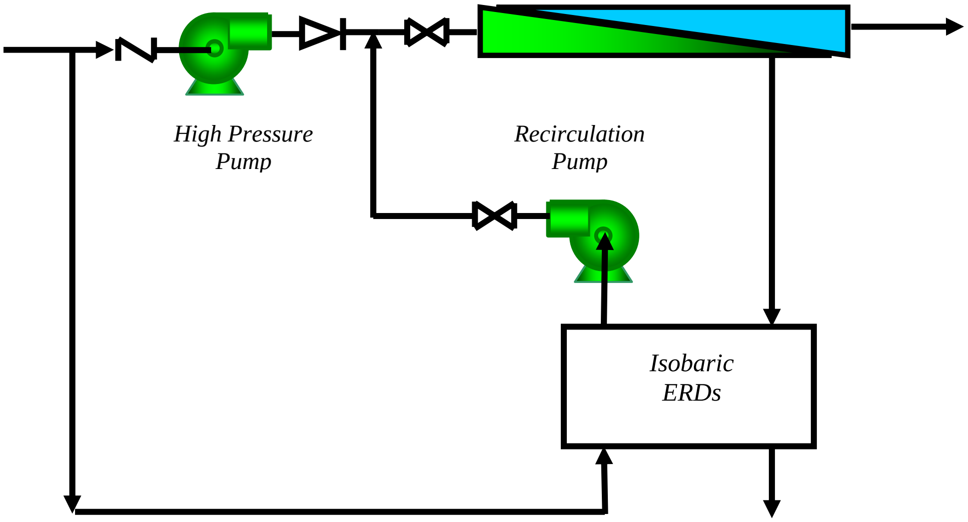


Figure 2. Reverse Osmosis with isobaric ERDs

The procedure necessary to design a reverse osmosis plant including a Pelton Turbine is currently well established. However, there are not available standard procedures for plants utilizing an isobaric ERD.

Our goal was to develop such procedures by providing a mathematical approach to understanding desalination plants using isobaric ERDs.

## II. RESEARCH CONDUCTED

### 2.1 General background

First, we determined the parameters that isobaric ERDs and desalination plants designs have in common. The parameters identified were flow, salinity and pressure.

We will use the following calculation nomenclature:

Flow → F
Salinity → S
Pressure → P
Recovery → R
$\Pi$ → osmotic pressure
$\eta_b$ → high pressure pump and recirculation pump efficiency
$\eta_m$ → high pressure motor and recirculation motor efficiency

Sea water → sw
Brine → br
Permeate → p
Input → i
Output → o
Membranes → m

### 2.2 Energy brine recovery with isobaric ERDs

If isobaric ERDs are represented as a black box, the system inputs and outputs could be illustrated as in figure 3.

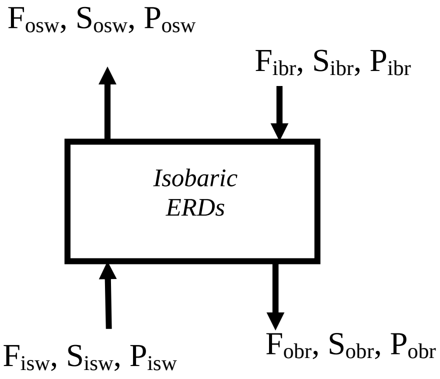


Figure 3. Isobaric ERDs parameters

Similarly, figure 4 illustrates the osmosis process inputs and outputs.

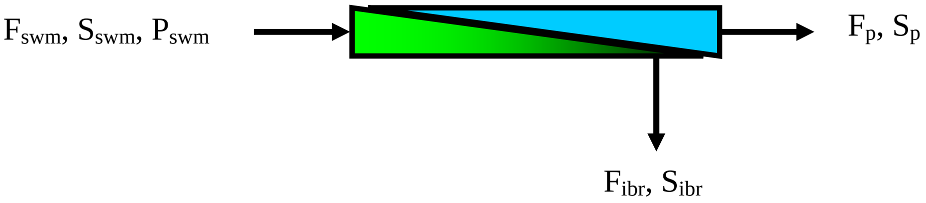


Figure 4. Reverse Osmosis parameters

The value of these inputs and outputs (figures 3,4) can be calculated using the equations provided in the following sections.

### *2.2.1 Flow calculation* – initial data: $R_m$ and $F_p$

$F_{ibr}$ and $F_{swm}$ are known and are function of $R_m$ and $F_p$, (1).

$$F_{ibr} = F_p \times (1/R_m - 1) \qquad F_{swm} = F_p / R_m \tag{1}$$

We have 3 unknown parameters: $F_{isw}$, $F_{osw}$ and $F_{obr}$ and 3 equations to determine them:

- Flow balance:

$$F_{isw} + F_{ibr} = F_{osw} + F_{obr} \tag{2}$$

- Overflush: Of. Overflush range is provided by isobaric ERDs manufactures.

$$Of = F_{isw} / F_{osw} - 1 \tag{3}$$

- Brine flow loss or Leak: L Leak is provided by isobaric ERDs manufactures as a function of temperature and brine input flow per isobaric ERDs unit.

$$L = F_{ibr} - F_{osw} \tag{4}$$

With (1), (2), (3) y (4) we can calculate $F_{isw}$, $F_{osw}$ and $F_{obr}$, see (5), (6) y (7)

$$F_{isw} = (Of + 1) \times (F_p \times (1/R_m - 1) - L) \tag{5}$$

$$F_{osw} = F_p \times (1/R_m - 1) - L \tag{6}$$

$$F_{obr} = F_p \times (1/R_m - 1) \times (Of + 1) - (Of \times L) \tag{7}$$

### *2.2.2 Salinity calculation* - $S_{isw}$ is known.

$S_p$ is a function of both reverse osmosis process and $S_{swm}$. Therefore, we need to iterate in order to achieve the accurate solution of the system. First of all we suppose a value of $S_{wm}$ and use this value to calculate $S_p$ using the membranes system design software.

$S_{obr}$ is a function of $F_{swm}$, $F_{isw}$, $F_p$, $F_{obr}$ and $S_p$ (8)

$$S_{obr} = (S_{isw} \text{ x } (F_{swm} + F_{isw} – F_{osw}) – F_p \text{ x } S_p)/F_{obr} \qquad (8)$$

Now we have 2 unknown parameters: $S_{osw}$ and $S_{ibr}$ and 2 equations to determine them:

- Salinity balance:

$$F_{isw} \text{ x } S_{isw} + F_{ibr} \text{ x } S_{ibr} = F_{osw} \text{ x } S_{osw} + F_{obr} \text{ x } S_{obr} \qquad (9)$$

- Mixing: Mix. Mix is provided by isobaric ERDs manufactures as a function of overflush.

$$\text{Mix} = (S_{osw} – S_{isw}) / (S_{ibr} – S_{isw}) \qquad (10)$$

With (9) and (10), we calculate $S_{osw}$ and $S_{ibr}$, see (11) and (12).

$$S_{ibr} = (F_{obr} \text{ x } S_{obr} - F_{isw} \text{ x } S_{isw} + F_{osw} \text{ x } S_{isw} \text{ x } (1 – \text{Mix}))/ (F_{ibr} – F_{osw} \text{ x Mix}) \qquad (11)$$

$$S_{osw} = S_{isw} \text{ x } (1 - \text{Mix}) + S_{ibr} \text{ x Mix} \qquad (12)$$

Then,

$$S_{swm} = R_m \text{ x } (S_p + S_{ibr} \text{ x } (1/R_m - 1)) \qquad (13)$$

Using the new value of $S_{swm}$ and the membranes system design software, we can then calculate the new value of $S_p$ and iterating using equations (8) to (13).

2.2.3 *Pressure calculation* - $P_{ibr}$ is known. $P_{ibr}$ and can be represented as a function of input pressure to membranes $P_{swm}$, pressure loss in the membranes ($\Delta_{p1}$) and pressure loss from the membranes output to the isobaric ERDs input ($\Delta_{p2}$), (14).

$$P_{ibr} = P_{swm}-(\Delta_{p1} + \Delta_{p2}) \qquad (14)$$

Isobaric ERDs manufacturer provide information about pressure loss in both sides of their equipments: the high pressure side ($\Delta p_3$) and the low pressure side ($\Delta p_4$).

$P_{isw}$ is known and can be a function of minimum pressure requirement at isobaric ERDs input of sea water or other system characteristics.

Using these parameters, we can calculate $P_{obr}$, see (15) and $P_{osw}$, see (16).

$$P_{osw} = P_{ibr}-\Delta_{p3} \qquad (15)$$

$$P_{obr} = P_{isw}-\Delta_{p4} \qquad (16)$$

There is one more isobaric ERDs requirement. The minimum pressure at isobaric ERDs output of brine ($P_{minobr}$). We must verify that $P_{obr} > P_{minobr}$.

*2.2.4 Energy consumption* – we can express energy consumption, Ec (Kwh/m3), as a function of the RO recovery ($R_m$), the Mix and the osmotic pressure of sea water ($\Pi_{isw}$). Additional parameters to consider include: a) the high pressure pump, b) the recirculation pump, c) the pressure losses in the membranes ($\Delta_{p1}$) and associated piping ($\Delta_{p5}$), d) the pressure losses in the high pressure side of isobaric ERDs ($\Delta_{p3}$) and the associated piping ($\Delta_{p6}$) and the previous equations.

$$Ec = 1/F_p \text{ x } (1/(\eta_b \text{ x } \eta_m)) \text{ x } (F_p \text{ x } (\Pi_{ibr}+\Delta_{p1}+\Delta_{p5}) + F_{ibr} \text{ x } (\Delta_{p1}+\Delta_{p3}+\Delta_{p5}+\Delta_{p6}) \qquad (17)$$

$$\Pi_{swm} = \Pi_{isw} \text{ x } (Mix \text{ x } (R_m-1)+1)/(1-Mix) \qquad (18)$$

Assuming $\Pi_p$~0 then

$$\Pi_{ibr} = \Pi_{swm} / (1-R_m) \qquad (19)$$

If

$$\Delta_{p1}+\Delta_{p5} = \Delta_{p7} \qquad (20)$$

$$\Delta_{p1}+\Delta_{p3}+\Delta_{p5}+\Delta_{p6} = \Delta_{p8} \qquad (21)$$

Then

$$Ec = (\Pi_{isw} \text{ x } (Mix \text{ x } (R_m-1)+1) / ((1-Mix)) \text{ x } (1-R_m)) + \Delta_{p7} + (1-R_m)/R_m \text{ x } \Delta_{p8})/ (\eta_b \text{ x } \eta_m) \qquad (22)$$

## III. RESULTS

According to our approach, the following plants local conditions must be known: the membranes manufacturers, the racks arrangement, the piping, the sea water salinity or the sea water temperature in order to determine the accurate flows, salinities and pressures of isobaric ERDs.

The proposed approached allows us to represent energy consumption as a function of known parameters such as the Mix, the $R_m$, the pressure losses and the sea water osmotic pressure.

In the section below we provide two applied examples of our approach:

CASE A: Estimation of the flows, salinities, pressures and energy consumptions of a desalination plant operating with an isobaric ERDs. In this case, we will compare two scenarios representing two modes of operation: one in which the plant is working with an overflush equal to 0%, and another in which the overflush is equal to 5%.

CASE B: Estimation of optimum RO recovery rate for the minimum energy consumption of the high pressure pump and the recirculation pump.

## 3.1 CASE A

We are going to compare two modes of isobaric ERDs performance.

### *3.1.1 Preliminary data*

The Plant characteristics are:

Production: 100.000 m3/day
RO Recovery rate: 45 %
Sea water temperature: 18ºC
Sea water salinity: 40.500 ppm
Level Plant: 5 m
Recovery System: isobaric ERDs
Number of pass: 1
Stages in first pass: 1
Units: 10 x 10.000 m3/day
System losses: according to the real mechanical and electrical equipments, piping, membranes, etc.
Overflush 0 % or 5 %

The study will include sea water pumps from level 0 to high pressure pumps, recirculation pumps and high pressure pumps.
It will include chemical pre treatment: ferric chlorine and antiscalant.
It will include physical pre treatment: pressure sand filters and cartridge filters.
It will not include the chemical membranes cleaning system.
It will not include filters washing system.
It will not include post treatment.
It will not include product pumps.

### *3.1.2 Study results*

Using the mathematical model and the preliminary data (section 3.1.1), we calculated the design of two plants. One of them with isobaric ERDs and Overflush = 0 and the other one with isobaric ERDs also but Overflush = 5%.

Figure 5 illustrates flow balance, salinity balance and energy consumption of a desalination plant with isobaric ERDs as brine energy recovery and overflush = 0 %.

Figure 6 illustrates Flow balance, salinity balance and energy consumption of a desalination plant with isobaric ERDs as brine energy recovery and overflush = 5 %.

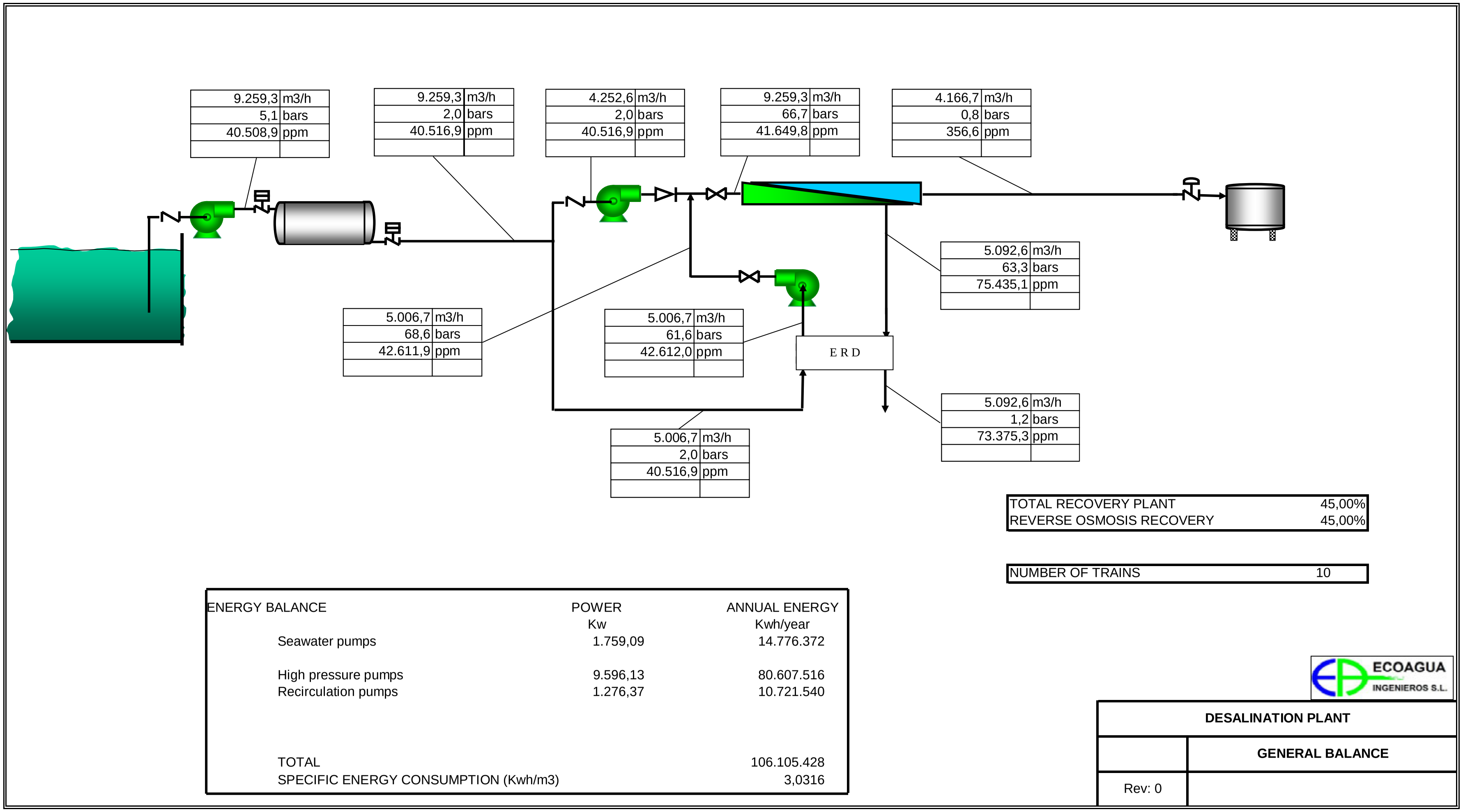
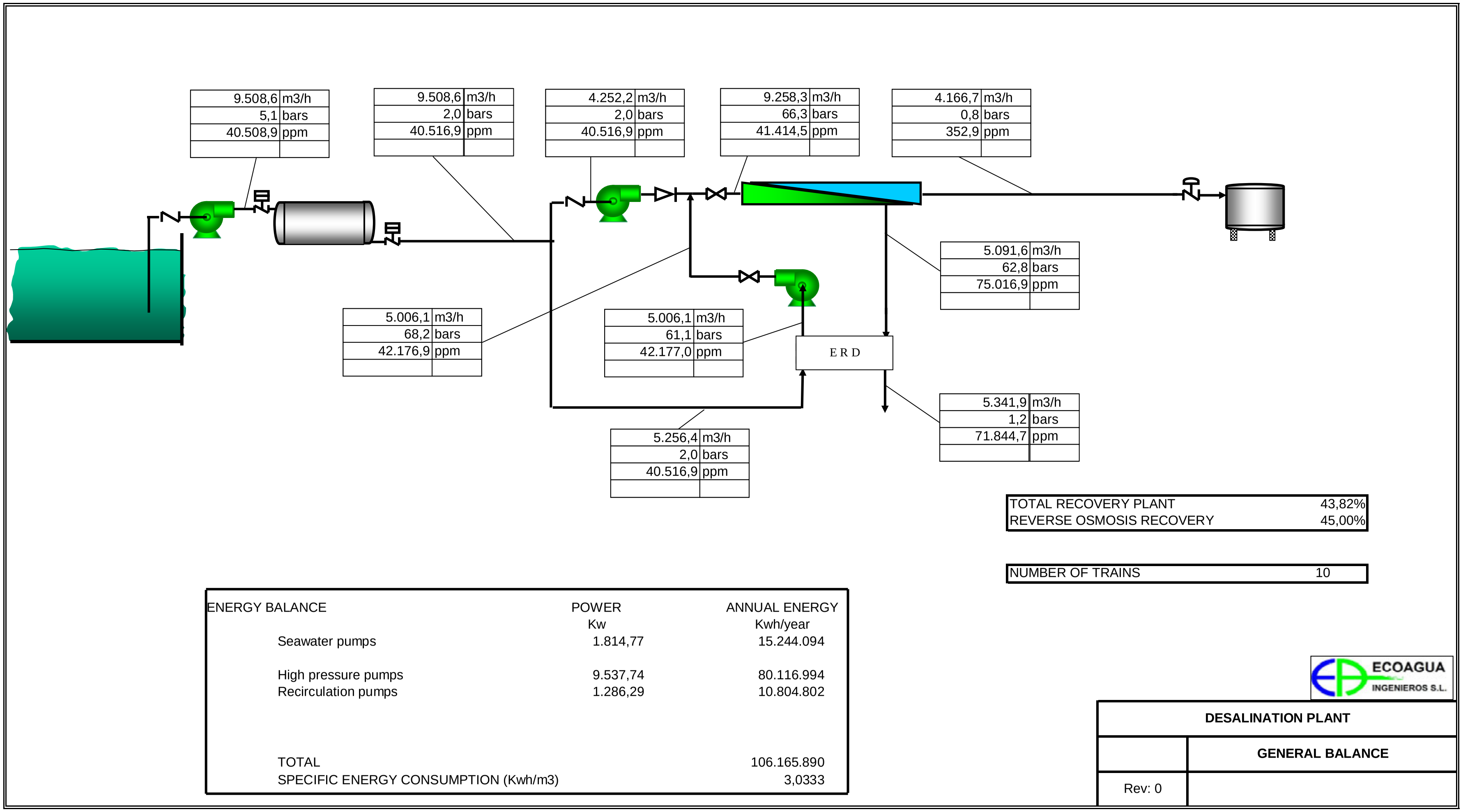


Figure 5. Flow balance, salinity balance and energy consumption of a desalination plant with isobaric ERDs and overflush = 0 %



| ENERGY BALANCE | POWER<br>Kw | ANNUAL ENERGY<br>Kwh/year |
|---|---|---|
| Seawater pumps | 1.814,77 | 15.244.094 |
| High pressure pumps | 9.537,74 | 80.116.994 |
| Recirculation pumps | 1.286,29 | 10.804.802 |
| TOTAL | | 106.165.890 |
| SPECIFIC ENERGY CONSUMPTION (Kwh/m3) | | 3,0333 |

Figure 6. Flow balance, salinity balance and energy consumption of a desalination plant with isobaric ERDs and overflush = 5 %

From figures 5 and 6 we can conclude:

1. Salinity and flow balance of the two plants are correctly estimated using this approach. If we had calculated separately isobaric ERDs and reverse osmosis process, the salinity balance and input pressure to membranes would not have been accurate.

2. Energy consumption of this desalination plant is higher with overflush =5% than with overflush=0%. This is because of the energy consumption of the sea water pumps and the recirculation pumps is higher.

3. If isobaric ERDs are working with an overflush not equal to zero then the plant recovery rate will not be equal to the RO recovery rate.

4. Working with overflush not equal to zero means working with higher feed flow of sea water or higher RO recovery.

## 3.2 CASE B

In this case, we will use our approach to determine the optimum RO recovery rate for minimum energy consumption.

According to section 2.2.4 energy consumption:

$$Ec = (\Pi_{isw} \text{ x } (Mix \text{ x } (R_m-1)+1) / ((1-Mix)) \text{ x } (1-R_m)) + \Delta_{p7} + (1-R_m)/R_m \text{ x } \Delta_{p8})/ (\eta_b \text{ x } \eta_m) \qquad (22)$$

$$\frac{\partial Ec}{\partial R_m} = \frac{1}{\eta_b \times \eta_m} \times \left( \frac{\Pi_{isw}}{1-Mix} \times \frac{1}{R_m{}^2 - 2 \times R_m + 1} - \frac{1}{R_m{}^2} \times \left( \Delta_{p8} \right) \right) \qquad (23)$$

$$\frac{\partial Ec}{\partial R_m} = 0 \qquad (24)$$

Thus, the optimum energy consumption (as a function of $R_m$) will be (25):

$$\left( \frac{\Pi_{isw}}{(1-Mix) \times (\Delta_{p8})} - 1 \right) x R_m{}^2 + 2 x R_m - 1 = 0 \qquad (25)$$

### *3.2.1 Preliminary data*

$\Pi_{isw}$ = 29,3 bar

Mix = 6 %

Range of possible pressure losses in the membranes, in the isobaric ERDs and in the associated piping ($\Delta_{p8}$) = [4,5-7,5] bar

### *3.1.2 Study results*

Figure 7 illustrates the optimum RO recovery rate as a function of $\Delta_{p8}$.

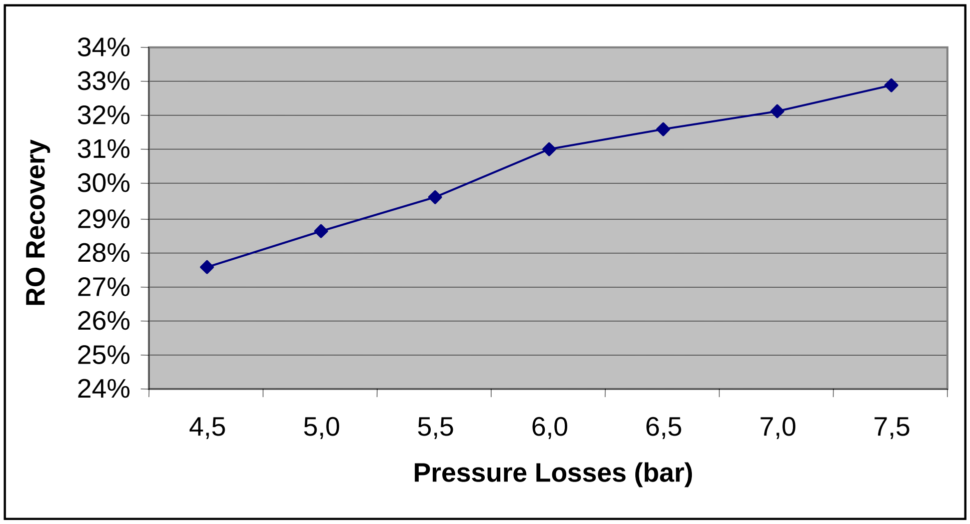


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Figure 7. Optimum RO recovery as a function of pressure losses in the membranes, isobaric ERDs and associated piping

This optimum RO recovery rate is calculated for the high pressure pump, the recirculation pump, the pressure losses in the membranes and associated piping and the pressure losses in the high pressure side of the isobaric ERDs and the associated piping. It is not included the rest of energy consumers of the plant, due to local conditions as the plant elevation above sea level, the pre treatment, the product pumps, etc. The optimum RO recovery including all plant energy consumers of a SWRO desalination plant with isobaric ERDs is displaced to higher values, [1] J.M Sanchez and R. Sanchez, 2006. In fact, we must consider each plant separately for achieving the optimum RO recovery. As a result, developing a single equation for characterizing isobaric ERDs efficiency has remained challenging.

## IV CONCLUSION

We have proposed an approach for estimating isobaric ERDs flow, salinity and pressure which is integrated with the rest of reverse osmosis plant calculation. This approach also considers correct salinity and flow balances. Independently, both the membranes system design software and the isobaric ERDs software allow obtaining an approximate solution to salinity and flow balances, but these estimates are not entirely correct.

Our proposed mathematical model (join with the rest of the plant design) achieves the goal of finding optimal performance and design of desalination plants using isobaric ERDs for brine energy recovery and provides an estimate of energy consumption of both processes RO and energy recovery, as a function of known parameters.

Our results show that we can not compare between different manufacturers of isobaric ERDs, for evaluating energy consumption of the plant, without taking into account plant parameters such as the RO recovery, the pressure losses, the local conditions or the reverse osmosis process. Therefore, comparisons of multiple isobaric ERDs required estimating the complete energy consumption of the plant for each case. In order to be able to evaluate an isobaric ERDs technically and economically we must considered the characteristics of both the isobaric ERD and the remaining plant processes at the same time.

## V LITERATURE CITED

1. J.M. Sanchez and R. Sanchez. *Representación y comparación entre las curvas de operación de un modelo de Planta Desaladora que utiliza como sistema de recuperación de energía SIP y Turbinas Pelton*. VI Congreso Nacional de AEDYR. Palma de Mallorca (Spain). November 2006.